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Analytical Study of Heat and Mass Transfer Enhancement in Free convection Flow with Chemical Reaction and Constant Heat Source in Nano-fluids

R.V.M.S.S Kiran Kumar[a,*] P. Durga Prasad[b], S.V.K. Varma[c]

[a,b,c]*Department of Mathematics, S.V. University, Tirupati-517502, A.P, India*

Abstract

The problem of unsteady MHD free convection flow of nanofluids through a porous medium bounded by a moving vertical semi-infinite permeable flat plate with constant heat source through porous medium is theoretically investigated. The velocity along the plate (slip velocity) is assumed to oscillate on time with a constant frequency and the temperature and concentration are assumed to be fluctuating with time harmonically from a constant mean at the plate. The analytical solutions of the boundary layer equations are solved by using the small perturbation approximations. Two types of nanofluids, namely *Cu*- water and Tio_2-water are used. The effects of various fluid flow and heat and mass transfer characteristics are discussed through graphs and tables.

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Keywords: Heat transfer; MHD; Porous medium; Nanofluid; Free convection; Chemical reaction.

1. Introduction

Convective heat transfer in nanofluids is a topic of major contemporary interest both in sciences and in engineering. Heating or cooling fluids such as water, ethylene, and glycol and engine oil play a crucial role in thermal management of hightech industries, but they have poor thermal characteristics, in particular thermal conductivity. Despite the considerable efforts to improve the rate of heat transfer by the usage of extended surfaces, mini-channels and micro channels, further enhancement in heating and cooling rate is always in demand. As solid materials possess

* Corresponding author. Tel.: +919493523133.
*E-mail address:*kksaisiva@gmail.com

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higher thermal conductivities many studies have been carried out on thermal properties of suspension of solid have been carried out on thermal properties of suspension of solid particles in convectional heat transfer fluids. Nanotechnology provides means to manufacture solid particles in nanometer scale. Choi[1] was the first to introduce the word nanofluid that represents the fluid in which nanoscaleparticles (diameter less than 50nm) are suspended in the base fluid. Nanoparticles are of great scientific interest as they are effectively a bridge between bulk materials and atomic or molecular structures. The common nanoparticles that have been used are aluminum, copper and titanium or their oxides. Experimental studies [2] shows that even with the small volumetric function of nanoparticles (usually less than 5%), the thermal conductivity of the base liquid can be enhanced by 5-20%. Nanofluids with or without the presence of magnetic field have many applications in the industries since materials of nanometer size have unique chemical and physical properties with regard to the sundry applications of nanofluids, the cooling applications of nanofluids include silicon mirror cooling, electronics cooling, vehicle cooling, transformer cooling, etc. This study is more important in industries such as hot rolling, melt spinning, extrusion, glass fiber production, wiredrawing, and manufacture of plastic and rubber sheets, polymer sheet and filaments, etc.The research on nanofluids is gaining a lot of attention in recent years. Abu-Nada [3] has state the application of nanofluids for heat transfer enhancement of separated flows encountered in a backward facing step.There is a vast amount of literature on the flow of nanofluidmodel proposed by Buongiorno[4].The study of heat transfer for an electrically conducting fluid past a porous plate under the influence of a magnetic field in a rotating frame of reference has attracted the interest of many industrial, astrophysical, geophysical, technological and engineering applications and many other practical applications, that is, in biomechanical problems. Philip [5] studied the Thermal properties of nanofluids. A review on hybrid nanofluids, Recent research, development and applications are studied by Sarkarn et al.[6]. Very recently,Venkateswaarlu and Satyanarayana[7] studied the chemical reaction and radiation absorption effects on the flow and heat transfer of a nanofluid in a rotating system.

Present paper deals the study to analyze the development of the unsteady free convection flow of a nanofluid past a moving vertical permeable semi-infinite flat plate in a rotating frame of reference. It is assumed that the plate is embedded in a uniform porous medium and oscillates in time with a constant frequency in the presence of a transverse magnetic field. So the main aim of this study is to extend the effort of reference Pop[8] et.al in two directions (i) the temperature and concentration are assumed to be fluctuating with time (ii) to consider the mass transfer effects.

2. Mathematical formulation of the problem

We consider an unsteady three dimensional free convection flow of an electrically conducting incompressible nanofluid past a semi-infinite vertical permeable moving plate embedded in a uniform porous medium in the presence of chemical reaction effect with constant heat source and fluctuating boundary conditions on velocity, temperature and concentration. We consider a Cartesian coordinate system $(\bar{x}, \bar{y}, \bar{z})$, the flow is assumed to be in the $\bar{x}$ -direction, which is taken along the plate and $\bar{z}$ -axis is normal to the plate. Also, it is assumed that the whole system is rotate with a constant velocity Ω about $\bar{z}$ -axis. We assume that the plate has an oscillatory movement on time $\bar{t}$ and frequency $\bar{n}$ with the velocity $\bar{u}(0,\bar{t})$, which is given by $\bar{u}(0,\bar{t}) = U_0[1+\varepsilon\cos(\bar{n}\bar{t})]$ where ε is a small constant parameter ($\epsilon \ll 1$) and U_0 is the characteristic velocity. Also, initially, the plate and the fluid are at same temperature T_∞' and concentration C_∞' in a stationary condition, when $t \geq 0$, the temperature and concentration at the plate fluctuates with time harmonically from a constant mean. A uniform external magnetic field B_0 is taken to be acting along $\bar{z}$ -axis. It is assumed that there is no applied voltage which implies the absence of an electric field. The fluid is a water based nanofluid containing two types of nanoparticles, either Cu (copper) or Tio_2 (titanium oxide). The nanoparticles are assumed to have a uniform shape and size. Moreover, it is assumed that both the fluid phase and nanoparticles are in thermal equilibrium state. Also assumed that the induced magnetic field is small compared to the external magneticfield. This implies a small magnetic Reynolds number for the oscillating plate. Using the relations between the density ρ_{nf} , viscosity μ_{nf} , thermal diffusivity α_{nf} , and heat capacitance $(\rho c_p)_{nf}$ of the nanofluids and base fluids are reported as (see Oztop and Abu-Nada) [9], and the thermo-physical properties of the base fluid (water), copper and titania which were used for code validation are given by Pop[8].

We assumed that the plate is semi-infinite plate so, the flow variables are functions of z and time t only. Following the nanofluid model proposed by Das and Tiwari [10] along with the Boussinesq and boundary layer approximations.

The basic equations in dimensionless form are given by

$$[1-\phi+\phi(\rho_s/\rho_f)](\frac{\partial u}{\partial t}-S\frac{\partial u}{\partial z}-Rv)=\frac{1}{(1-\phi)^{2.5}}\frac{\partial^2 u}{\partial z^2}+[1-\phi+\phi((\rho\beta)_s/(\rho\beta)_f)]\theta-(M+\frac{1}{K})u \quad (1)$$

$$[1-\phi+\phi(\rho_s/\rho_f)](\frac{\partial v}{\partial t}-S\frac{\partial v}{\partial z}+Ru)=\frac{1}{(1-\phi)^{2.5}}\frac{\partial^2 u}{\partial z^2}-(M+\frac{1}{K})v \quad (2)$$

$$[1-\phi+\phi((\rho c_p)_s/(\rho c_p)_f)](\frac{\partial \theta}{\partial t}-S\frac{\partial \theta}{\partial z})=\frac{1}{\Pr}[\frac{k_{nf}}{k_f}\frac{\partial^2\theta}{\partial z^2}-Q_H\theta] \quad (3)$$

$$\frac{\partial\psi}{\partial t}-S\frac{\partial\psi}{\partial z}=\frac{1}{Sc}\frac{\partial^2\psi}{\partial z^2}-Kr\psi \quad (4)$$

The corresponding dimensionless boundary conditions are written as follows

$$u(z,t)=0,\ v(z,t)=0,\ \theta(z,t)=0,\psi(z,t)=0 \quad for\ \ t<0\ \ and\ any\ \ z$$
$$u(0,t)=1+\frac{\varepsilon}{2}\{e^{\text{int}}+e^{-\text{int}}\},v(0,t)=0,\theta(0,t)=1+\frac{\varepsilon}{2}\{e^{\text{int}}+e^{-\text{int}}\},$$
$$\psi(0,t)=1+\frac{\varepsilon}{2}\{e^{\text{int}}+e^{-\text{int}}\},u(\infty,t)\to 0,v(\infty,t)\to 0,\theta(\infty,t)\to 0,\psi(\infty,t)\to 0 \quad for\ \ t\geq 0 \quad (5)$$

In the above dimensionless equations we use the following dimensionless variables:

$$z=(U_0/v_f)\bar{z},\ t=(U_0^2/v_f)\bar{t},\ n=(v_f/U_0^2)\bar{n},\ u=(\bar{u}/U_0).$$

$$v=\bar{v}/U_0,\theta=(T-T_\infty')/(T_w'-T_\infty'),\ \psi=(C-C_\infty')/(C_w'-C_\infty').$$
$$S=\frac{w_0}{U_0},M=\frac{\sigma B_0^2 v_f}{\rho_f U_0^2},R=\frac{2v_f\Omega}{U_0^2},\ Q_H=\frac{Qv_f^2}{k_f U_0^2},Kr=\frac{K_l v_f}{U_0^2},\ Sc=\frac{v_f}{D_B},K=\frac{k\rho_f u_0^2}{v_f^2} \quad (6)$$

Here $\Pr=\frac{v_f}{\alpha_f}$ is the Prandtl number, S is the suction *(S>0)* or injection *(S<0)* parameter, M is the Magnetic parameter, R is the Rotation parameter, Q_H is the heat source parameter, Kr is the chemical reaction parameter , and Sc is the Schmidt number, K is the permeable parameter, U_0 is the uniform reference velocity, Ω is the constant rotation velocity, σ is the electric conductivity, w_0 is the mass flux velocity, Q is the additional heat source, k_f is the thermal conductivity of the base fluid, ρ_f is the fluid density, and v_f is the kinematic viscosity of the fluid part of the nanofluid,

Using Eq.(1), the velocity characteristic U_0 is defined as

$$U_0=[g\beta_f(T_w'-T_\infty')v_f]^{1/3}.$$

Now, in order to obtain the desired solutions of Eqs.(1)-(4), we assume the fluid velocity in the complex form as

$$\chi(z,t)=u(z,t)+iv(z,t) \quad (7)$$

By using Eq.7 we can simplify Eqs.(1) and (2) to the following equation

$$(1-\phi+\phi(\rho_s/\rho_f))[\frac{\partial\chi}{\partial t}-S\frac{\partial\chi}{\partial z}+iR\chi]=\frac{1}{(1-\phi)^{2.5}}\frac{\partial^2\chi}{\partial z^2}+[1-\phi+\phi(\rho\beta)_s/(\rho\beta)_f]\theta-(M+\frac{1}{K})\chi \quad (8)$$

The boundary conditions (5) become

$$\begin{aligned}&\chi(z,t)=0,\theta(z,t)=0,\psi(z,t)=0,\ for\ t<0\ and\ any\ z\\&\chi(0,t)=1+\frac{\varepsilon}{2}(e^{\mathrm{int}}+e^{-\mathrm{int}}),\ \theta(0,t)=1+\frac{\varepsilon}{2}(e^{\mathrm{int}}+e^{-\mathrm{int}}),\ \psi(0,t)=1+\frac{\varepsilon}{2}(e^{\mathrm{int}}+e^{-\mathrm{int}}),\\&\chi(\infty,t)\to 0,\theta(\infty,t)\to 0,\psi(\infty,t)\to 0,\ for\ t\geq 0\end{aligned} \tag{9}$$

2. Method of solution

In order to solve the above system of partial differential Eqs.(1)-(4) under the boundary conditions (5) in the neighborhood of the plate, we assume that [11].

$$\chi(z,t)=\chi_0(z,t)+\frac{\varepsilon}{2}\{e^{\mathrm{int}}\chi_1(z,t)+e^{-\mathrm{int}}\chi_2(z,t)\}, \tag{10}$$

$$\theta(z,t)=\theta_0(z,t)+\frac{\varepsilon}{2}\{e^{\mathrm{int}}\theta_1(z,t)+e^{-\mathrm{int}}\theta_2(z,t)\}, \tag{11}$$

$$\psi(z,t)=\psi_0(z,t)+\frac{\varepsilon}{2}\{e^{\mathrm{int}}\psi_1(z,t)+e^{-\mathrm{int}}\psi_2(z,t)\} \tag{12}$$

For($\epsilon \ll 1$), then substituting Eqs.(10)-(12) into Eqs.(3) ,(4) , (8) and (9), equating the harmonic and non-harmonic terms and neglecting the higher order terms of $O(\varepsilon^2)$, using the relevant boundary conditions, we obtain the expressions for velocity, temperature and concentration as .

$$\chi(z,t)=(1+B_1)e^{-m_7 z}-B_1e^{-m_4 z}+\frac{\varepsilon}{2}[((1+B_2)e^{-m_8 z}-B_2e^{-m_5 z})e^{\mathrm{int}}+((1+B_3)e^{-m_9 z}-B_3e^{-m_6 z})e^{-\mathrm{int}}] \tag{13}$$

$$\theta(z,t)=e^{-m_4 z}+\frac{\varepsilon}{2}[e^{-m_5 z}e^{\mathrm{int}}+e^{-m_6 z}e^{-\mathrm{int}}] \tag{14}$$

$$\psi(z,t)=e^{-m_1 z}+\frac{\varepsilon}{2}[e^{-m_2 z}e^{\mathrm{int}}+e^{-m_3 z}e^{-\mathrm{int}}] \tag{15}$$

The physical quantities of engineering interest are Skin-friction coefficient C_f, the rate of heat transfer coefficient in terms of Nusselt number Nu, and the rate of mass transfer coefficient in terms of Sherwood number Sh, which are defined as follows

$$C_f=\frac{\overline{\tau_w}}{\rho_f U_0^2}=\frac{1}{(1-\phi)^{2.5}}\chi'(0)\ =\frac{1}{(1-\phi)^{2.5}}\{-m_7(1+B_1)+m_4B_1+\frac{\varepsilon}{2}[e^{\mathrm{int}}((-m_8(1+B_2))+m_5B_2)+e^{-\mathrm{int}}(-m_9(1+B_3))+m_6B_3]\} \tag{16}$$

The local Nusselt number Nu is given by

$$Nu=\frac{\bar{x}\,\overline{q_w}}{k_f(T_W-T_\infty)}=-\frac{k_{nf}}{k_f}\theta'(0)\ =\frac{k_{nf}}{k_f}[m_4+\frac{\varepsilon}{2}(m_5e^{\mathrm{int}}+m_6e^{-\mathrm{int}})] \tag{17}$$

The local Sherwood number Sh_x is given by

$$Sh_x=\frac{\bar{x}\,\overline{q_w}}{D_B(C_w-C_\infty)}=-x\psi'(0)=-m_1+\frac{\varepsilon}{2}[-m_2e^{\mathrm{int}}-m_3e^{-\mathrm{int}}] \tag{18}$$

Where $\bar{\tau}$, $\overline{q_w}$ and $\overline{q_m}$ are the wall shear stress or skin friction, the wall heat flux and the wall mass flux from the plate respectively, which are given by

$$\bar{\tau}=\mu_{nf}\left(\frac{\partial\bar{u}}{\partial\bar{z}}\right)_{\bar{z}=0},\ \overline{q_w}=-k_{nf}\left(\frac{\partial T}{\partial\bar{z}}\right)_{\bar{z}=0},\overline{q_m}=\mu_{nf}\left(\frac{\partial C}{\partial\bar{z}}\right)_{\bar{z}=0}. \tag{19}$$

Where the constants $B_1,B_2,B_3,m_1,m_2,m_3,m_4,m_5,m_6,m_7,m_8,m_9$ are not mentioned due to brevity.

4. Results and Discussion

The analytical solutions are performed for concentration, temperature and velocity profiles for various values of physical parameters such as suction parameter (S), heat generation parameter (Q_H), Chemical reaction parameter

(Kr), Rotation parameter (R), Magnetic field parameter (M), and Permeability parameter (K) are depicted graphically from Figure1 to 6. Throughout the calculations the parametric values are chosen as $\varepsilon = 0.1, t = 1, n = 10, \Pr = 0.71$. All the graphs therefore correspond to these values unless specially indicated on the appropriate graph.

Fig.1indicates the nanofluid concentration profiles for various values of chemical reaction parameter Kr. It is found that an increase in Kr causes to decrease the nanofluid species concentration as well as it decreases the boundary layer thickness.Fig.2(a),(b) illustrates the effects of nanoparticle volume fraction for $S = 0$ and $S = 1.5$ for the nanoparticles Cu and Tio_2 .From these figures it is clear that an increase of the nanoparticle volume fraction leads to the increase of the thermal boundary layer thickness. Also, the thickness of the thermal boundary layer for $Cu-$ water is greater than that of pure water $(\phi = 0)$. This is because copper has high thermal conductivity and its addition to the water based fluid increases the thermal conductivity for the fluid, so the thickness of the thermal boundary layer increases and hence it can be observed that the thermal boundary layer thickness becomes thinner in the case of suction S > 0. The influence of heat generation parameter Q_H on the nanofluid temperature profiles for Cu and Tio_2 nanoparticles are shown in Fig.3 (a) and (b).From these figures, it is observed that the increasing in Q_H for $\phi = 0$ (regular fluid) and $\phi \neq 0$ (nanofluids) causes to decrease the nanofluid temperature profiles. This is due to the fact that when heat is absorbed, the buoyancy forces decreases, which retard the flow rate and thereby give rise to a decrease in the temperature profiles of nanoparticles. The graphical representation of the velocity profiles χ for different values of the Cu nanoparticles volume fraction in absence of magnetic field *(M=0)*and in presence of magnetic field*(M=10)*when ϕ=0, 0.5, 0.15are shown in Fig.4.These results are good agreement with Pop[8].The velocity profiles for various values of rotation parameter R for the nanoparticles Cu and Tio_2 for $\phi = 0$ (regular fluid) and $\phi \neq 0$ (nanofluids) are plotted in figures 5(a) and (b). From these figure it is observed that the velocity decreases as R increases. The rotation of Cu nanoparticles is faster than that of Tio_2 nanoparticles. Fig.6 depicts the velocity profiles for different values of permeability parameter K with nanoparticles Cu. It is noticed that as K increases, the nanofluid velocity also increases. Physically, this means that the porous medium impact on the boundary layer growth is significant due to the increase in the thickness of the momentum boundary layer.

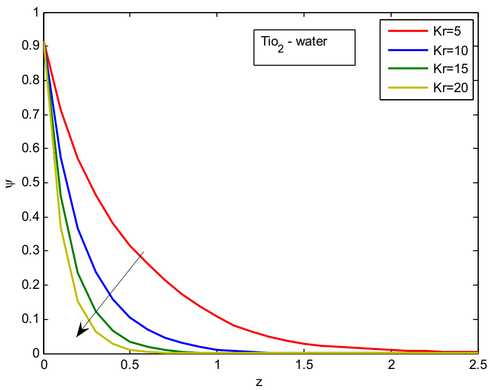

Fig:1 Concentration Profiles for different values of Kr with $S = 0.5, Sc = 0.22$.

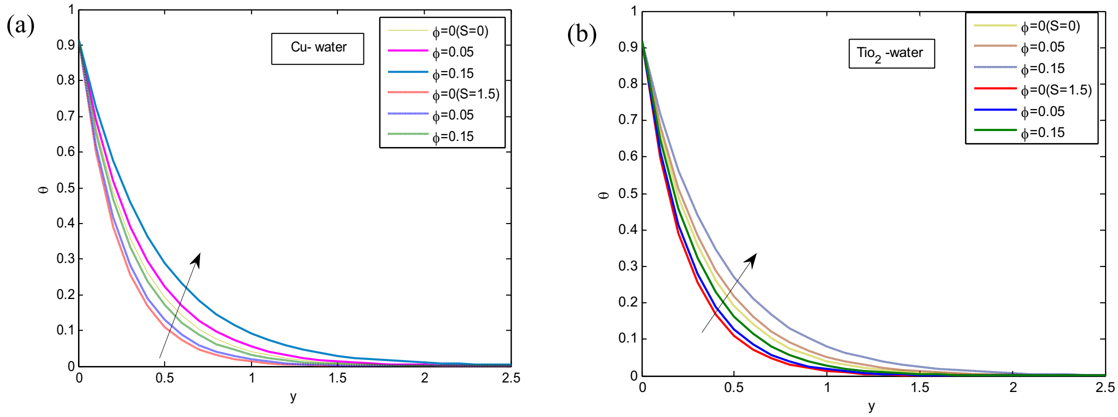

Fig: 2 Temperature Profiles for different values of ϕ in the absence/presence of suction effect with $R = 0.1, Q_H = 10, M = 0.4.$

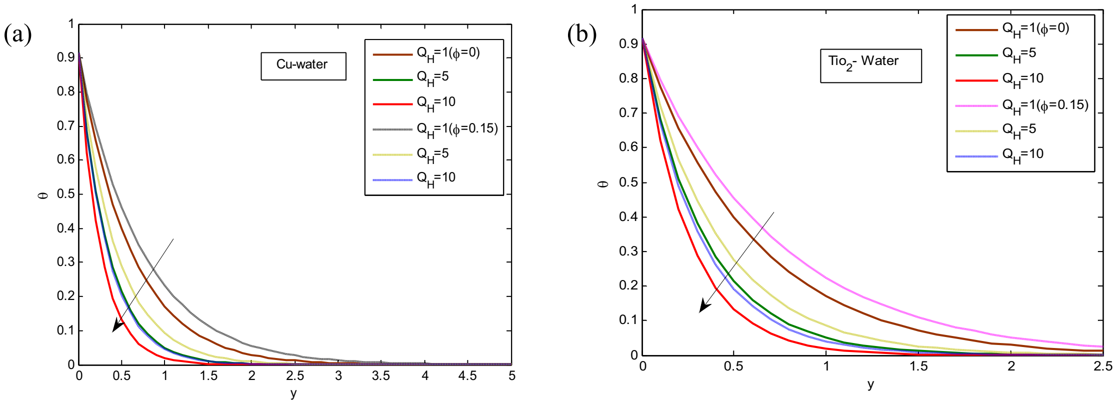

Fig.3 Temperature Profiles for various values of ϕ in the absence/presence of Heat generation parameter Q_H with $S = 1, R = 0.2, K = 0.5.$

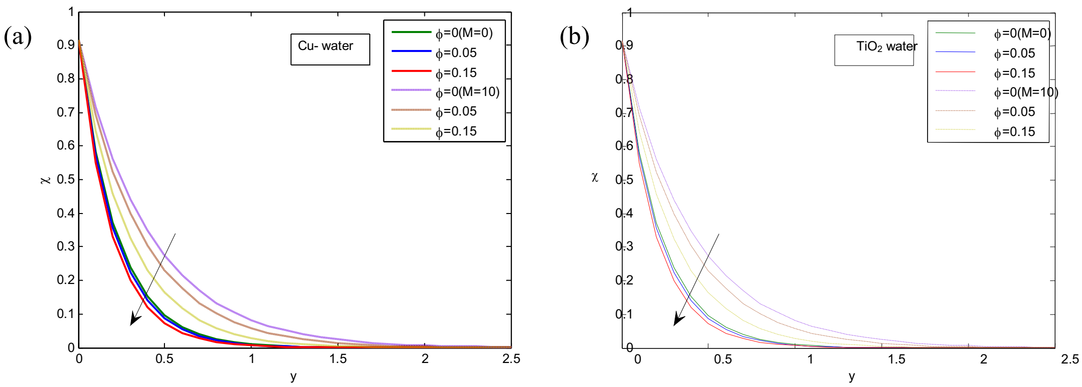

Fig.4 Velocity Profiles for different values of ϕ in the absence/presence of M with $S = 1.5, Q_H = 10; M = 0.4, K = 0.5, \phi = 0.15.$

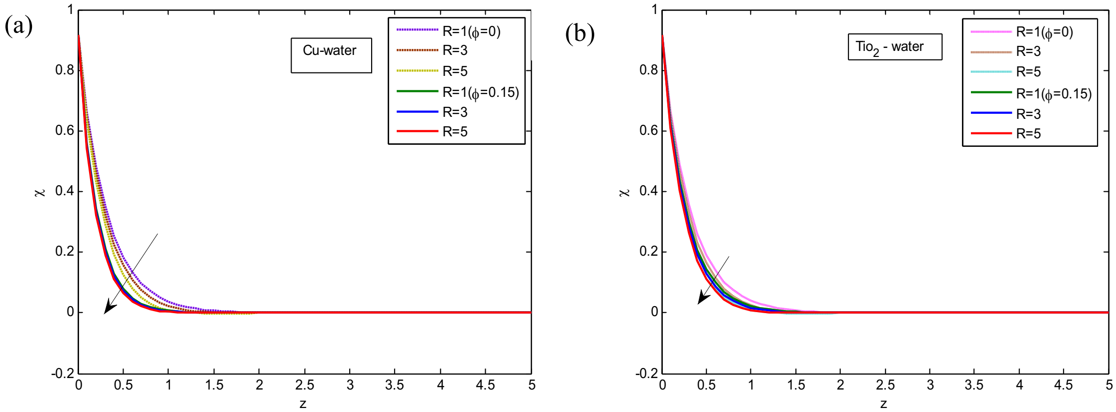

Fig.5Velocity Profiles for different values of ϕ in the absence/presence of R with $Pr = 6.72, S = 1, Q_H = 10, M = 0.5, R = 0.5, K = 0.5.$

Table 1: Numerical values of Sherwood number (Sh)

Sc	S	Kr	Sh
0.22	1.5	0.5	0.7432
0.60	1.5	0.5	2.0270
0.78	1.5	0.5	2.6351
0.22	**2**	0.5	0.8440
0.22	**4**	0.5	1.2471
0.22	**6**	0.5	1.6502
0.22	1.5	**5**	2.5571
0.22	1.5	**10**	4.5725
0.22	1.5	**15**	6.5879

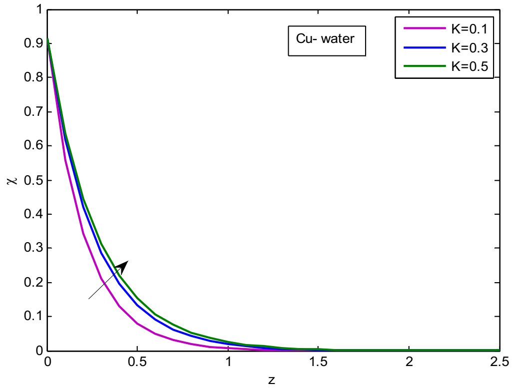

Fig. 6 Velocity Profiles for different values of K when $S = 1, Q_H = 10, M = 0.5, R = 0.5.$

Table 2: Numerical values of Nusselt number (Nu).

Q_H	ϕ	R	S	Nu
1	0.15	0.3	1.5	3.1115
5	0.15	0.3	1.5	4.6437
10	0.15	0.3	1.5	5.7873
10	**0.01**	0.3	1.5	4.0525
10	**0.03**	0.3	1.5	4.2530
10	**0.05**	0.3	1.5	4.4659
10	0.15	**1**	1.5	5.7873
10	0.15	**3**	1.5	5.7873
10	0.15	**5**	1.5	5.7873
10	0.15	0.3	**2**	6.3957
10	0.15	0.3	**4**	8.8885
10	0.15	0.3	**6**	11.4684

Table 3: Numerical values of Skin-friction coefficient (*Cf*)

Q_H	ϕ	R	S	K	M	Cf
1	0.15	0.3	1.5	0.5	0.4	-5.5793
5	0.15	0.3	1.5	0.5	0.4	-1.3586
10	0.15	0.3	1.5	0.5	0.4	-7.5121
10	**0.01**	0.3	1.5	0.5	0.4	-3.0833
10	**0.03**	0.3	1.5	0.5	0.4	-3.5177
10	**0.05**	0.3	1.5	0.5	0.4	-4.0024
10	0.15	**1**	1.5	0.5	0.4	-7.4730
10	0.15	**3**	1.5	0.5	0.4	-7.3093
10	0.15	**5**	1.5	0.5	0.4	-7.2866
10	0.15	0.3	**2**	0.5	0.4	-10.4478
10	0.15	0.3	**4**	0.5	0.4	-15.2809
10	0.15	0.3	**6**	0.5	0.4	-23.8407
10	0.15	0.3	1.5	**1**	0.4	-7.0060
10	0.15	0.3	1.5	**1.5**	0.4	-6.8423
10	0.15	0.3	1.5	**3**	0.4	-6.6767
10	0.15	0.3	1.5	0.5	**0**	-7.3042
10	0.15	0.3	1.5	0.5	**2**	-8.6140
10	0.15	0.3	1.5	0.5	**5**	-4.8261

The numerical values of Sherwood number Sh for different flow parameters Sc, S, Kr are presented in Table 1. From this table, one can be noted that the Sherwood number Sh increases with an increasing values of Sc, S and Kr for nanofluids $(\phi \neq 0)$ with nanoparticles Cu .The effects of $Q_H, \phi, and\ S$ on Nusselt number Nu are shown in Table 2**.** It is clear that the rate of heat transfer coefficient Nu increases as Q_H ,ϕ and S for nanofluids $(\phi \neq 0)$ with nanoparticles Cu . Also it is observed that there is a considerable effect of suction parameter S on Nu .Table 3 denote the numerical values of Skin-friction coefficient for different fluid flow parameters $Q_H, \phi, R, S, K, and\ M$. From this table it is clear that the skin friction coefficient Cf increases with R and K .While it increases upto $Q_H = 5$ then suddenly decreases for $Q_H = 10$ .Also Cf is decreases upto $M = 2$, while it is increases significantly and hence Cf decreases with an increasing values of ϕ .

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